**Notes: Week of Sept14.Fall2012**

Course website: [www.cis.syr.edu/~sueo/cis275](http://www.cis.syr.edu/~sueo/cis275)

**Interpretations:**

-for interpretations, pick a set, values of variables, and values of predicates to give statements certain truth values. These can be numbers, colors, anything really.

-Usually using the empty set won’t be allowed when trying to give something a specific truth value

**Logical Equivalence**

Two predicate-logic formulas are logically equivalent when each has the same truth values as the other regardless of the choice of quantified sets or the interpretations of the predicates

**How to show two formulas are not logically equivalent:**

* Come up with a specific set of interpretations that gives the formulas different truth values

**How to show that an argument form is invalid:**

* Find a specific set and interpretations that make the premises both true and the conclusion false

**Existential Generalization**

If **p(a) and a ∈ *U*** are both true, then you can conclude:

**∃x ∈ *U* such that p(x)**

**Existential Instantiation**

If the premise **∃z ∈ *U* such that p(z)** is true,

And **a** is a fresh variable name (it has not been used previously in the proof),

Then you can conclude:

**p(a)**

**[see slides for specific examples]**

**Not in slides**:

|  |  |
| --- | --- |
| **Universal Instantiation**   * No constraints   If ∀x ∈ *U*, p(x) and a ∈ *U* ,  Then p(a) is true. | **Existential Generalization**   * No constraints   If p(a) and a ∈ *U,*  Then ∃x ∈ *U* such that p(x) is true |
| **Universal Generalization**   * **Constraint:** must ensure name ‘a’ arbitrary   If p(a) and a is arbitrary element of U  Then ∀x ∈ *U*,p(x)is true | **Existential Instantiation**   * **Constraint:** must ensure that name ‘a’ is fresh   ∃x ∈ *U* such that p(x) is true  And a is a fresh name for item a ∈ *U*  Then p(a) is true |

**More on Sets:**

* Definitions:

Set Equality: Sets A and B whose elements come from some universe *U* are equal (A=B) if and only if:

∀x ∈ *U,* x ∈ A ↔x ∈ B

(all elements in A are in B and all elements in B are in A)

Subset: and A is a subset of B (written A⊆B) if and only if

∀x ∈ *U,* ∈ A →x ∈ B

(all elements in A are in B)

Proper subset: A subset is a proper subset (written A ⊂ B) If B contains at least one element that A does not

Empty set – {}, or ∅ ,a set with no elements; The empty set is a subset of all sets

* Basic Set Operations:

Ac – the complement of A. The set of all elements not in A

Ac = {x : x ∉ A }

A ∩ B – The intersection of A and B; contains all elements in both sets A and B

A ∩ B = {x : x ∈ A and x ∈ B }

A ∪ B – The union of A and B; contains all items that are in A or in B (or both)

A ∪ B = {x : x ∈ A or x ∈ B }

A \ B – The set difference of A and B (also A – B); items in A that are not in B

A \ B = {x : x ∈ A and x ∉ B }

Disjoint – Two sets are disjoint if they share no elements:

Iff A ∩ B is the empty set

A x B – The cartesian product of A and B, The set of all pairs of elements from A and B:

A x B = { (a,b) : a ∈ A and b ∈ B }

Is A={1,2} B ={1,2,3}, Ax B = { (1,1), (1,2), (1,3), (2,1), (2,2), (2,3) }

P (A) – the power set of A (written P (A) with a script P); The set of all subsets of A

P(A) = {B : B ⊆ A }

Cardinality – The number of elements of a set, written |A|. ie: |{a,b,c}|= 3

Finite – A is finite if and only if |A| is a natural number

Infinite – pretty straightforward, an infinite set is a set that is not finite